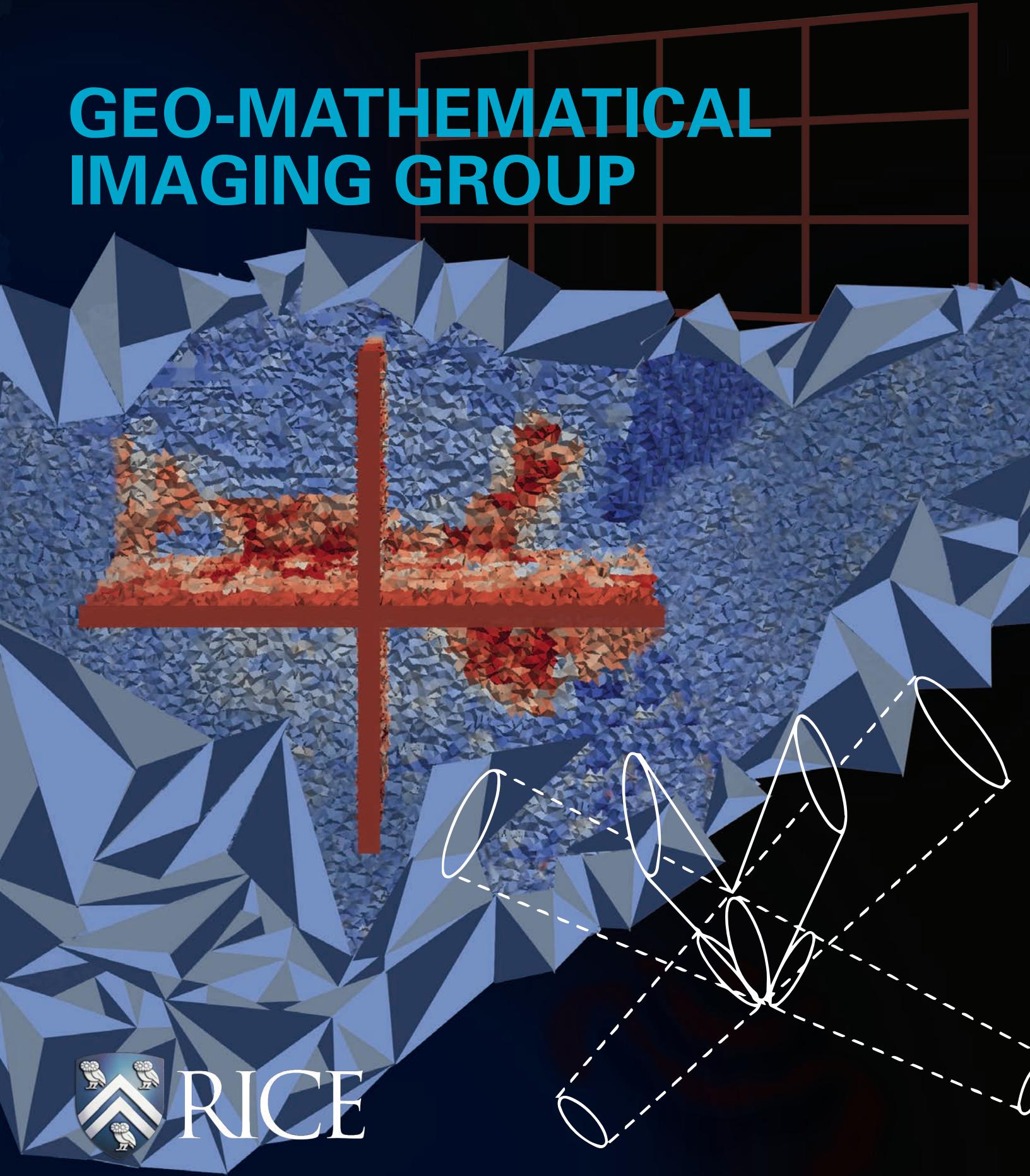




GEO-MATHEMATICAL IMAGING GROUP



RICE



GEO-MATHEMATICAL IMAGING GROUP

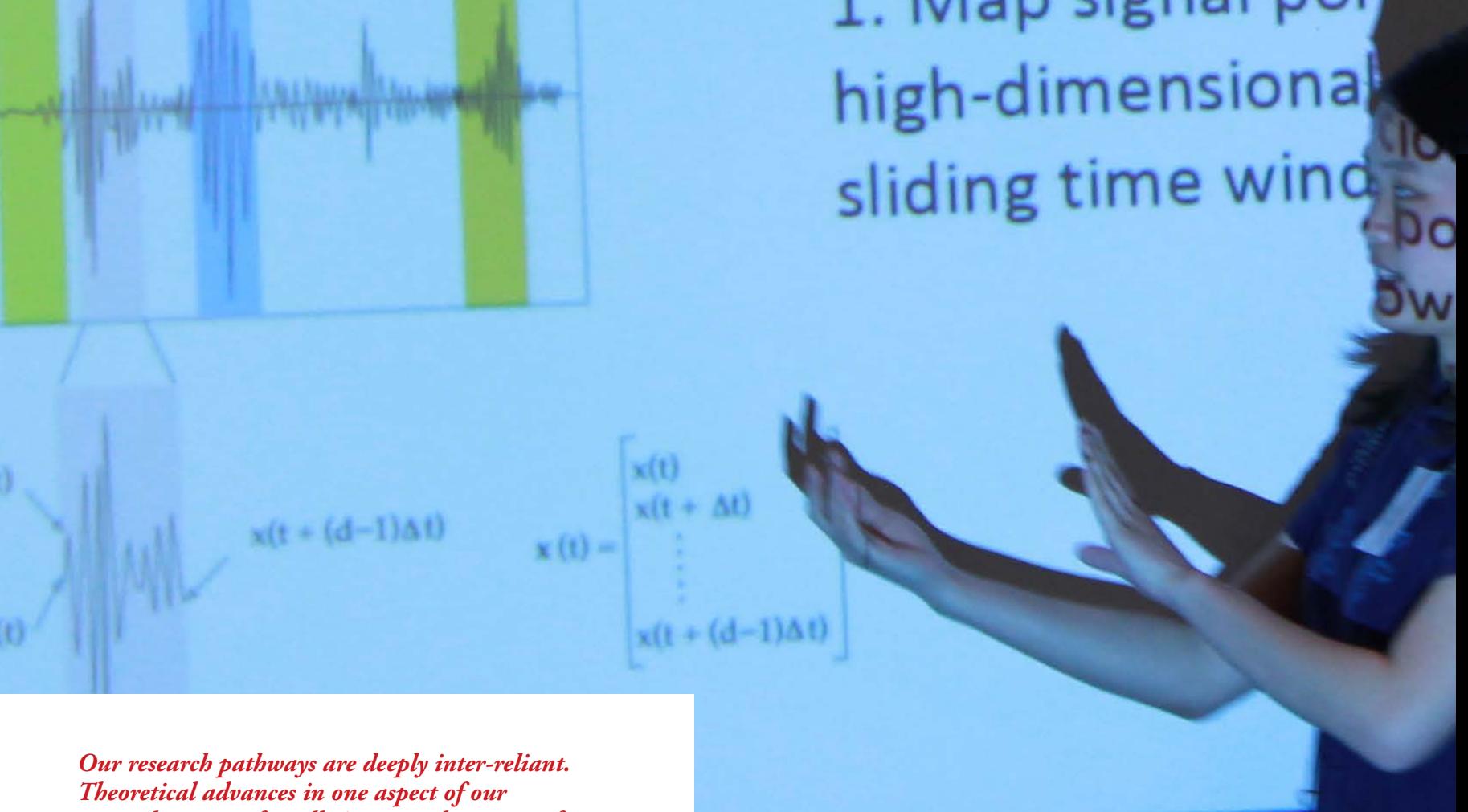
The cutting edge of seismology research

The Geo-Mathematical Imaging Group is a Joint University-Industry Project funded by collaborators in the Energy and High-Performance Computing industries. Based at Rice University in Houston, GMIG is at the forefront of research into cutting edge solutions in geophysical imaging and computational seismology. Our program spans the full seismological discipline with research in theory, analysis and massively parallel computation. Foundational theoretical investigations are interwoven with algorithmic development, data processing experimentation and machine learning projects, with the ultimate goal of extraction of features from the sub-surface. Headed by geo-mathematician Prof. Maarten de Hoop with collaborating faculty at Rice and globally, GMIG benefits from our prolific inter-institutional collaborations, through which we incorporate the most recent progress in earthquake dynamics, global seismology, monitoring, new sensors. In our research program different areas of mathematics meet, including the analysis of PDEs, microlocal analysis, harmonic analysis, differential geometry, algebraic geometry and numerical linear algebra. In recent years, our portfolio has evolved to encompass the foundations of deep learning connected to nonlinear (seismic) inverse problems, monitoring and forecasting.

We apply investigative routes in:

- analysis of inverse problems in exploration seismology and geodesy with impact on (approximate) reconstruction strategies and UQ
- underlying physics and signals theory affecting the design of information-content driven acquisition
- advancement of deep learning and artificial intelligence in the context of the full range of inverse problems, data acquisition, and algorithmic design
- research and development in extreme-scale computing and optimal design of parallel computational pathways





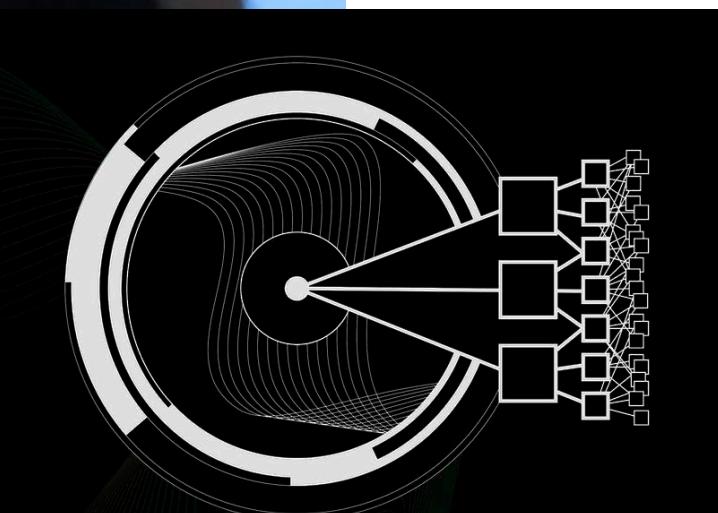
Our research pathways are deeply inter-reliant. Theoretical advances in one aspect of our research may profoundly impact other areas of our investigations. Our global network of active collaborations with leading researchers in diverse fields ensures we have access to pertinent advances immediately.

GMIG applies cutting-edge developments across multiple fields through:

- Ongoing research visits and exchanges with collaborators
 - Organization of, and attendance at relevant conferences globally
 - Hosting of the annual Math + X Symposium, where we gather together leading scientists in mathematics, geophysics, machine learning and data science
 - Collaborative research projects
- Access to our researchers for direct consultation;
 - Access to highly skilled internship candidates;
 - Twice-yearly research workshops updating and teaching the latest results of our work;
 - Exclusive access to code resulting from our research;
 - Early access to research in advance of publication.

Please see our Projects & Collaborations map for an overview of our global collaborations.

Industry sponsors of GMIG gain direct benefits of co-directed projects. Sponsors are invited to bring problems to the table for in-depth studies. Sponsors gain benefits of:



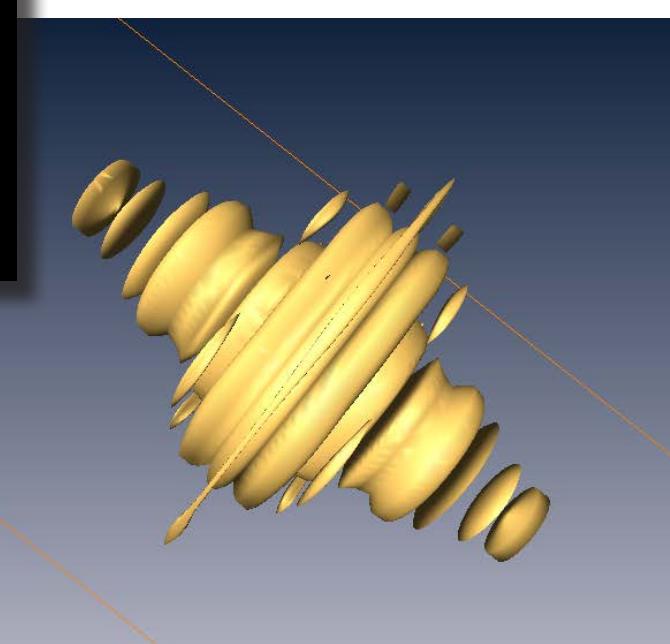
MATH + X SYMPOSIUM

With generous support from the Simons Foundation for Advancing Research in Mathematics and the Basic Sciences, we hosted the 2018 Math + X Symposium in Data Science and Inverse Problems in Geophysics, a vibrant assembly of leading minds in geoscience research. The Symposium marked a restructuring of our research program, with a significantly expanding effort in the analysis of deep learning through numerous new collaborations. New directions of research will apply deep learning techniques to nonlinear inverse problems, monitoring and forecasting.

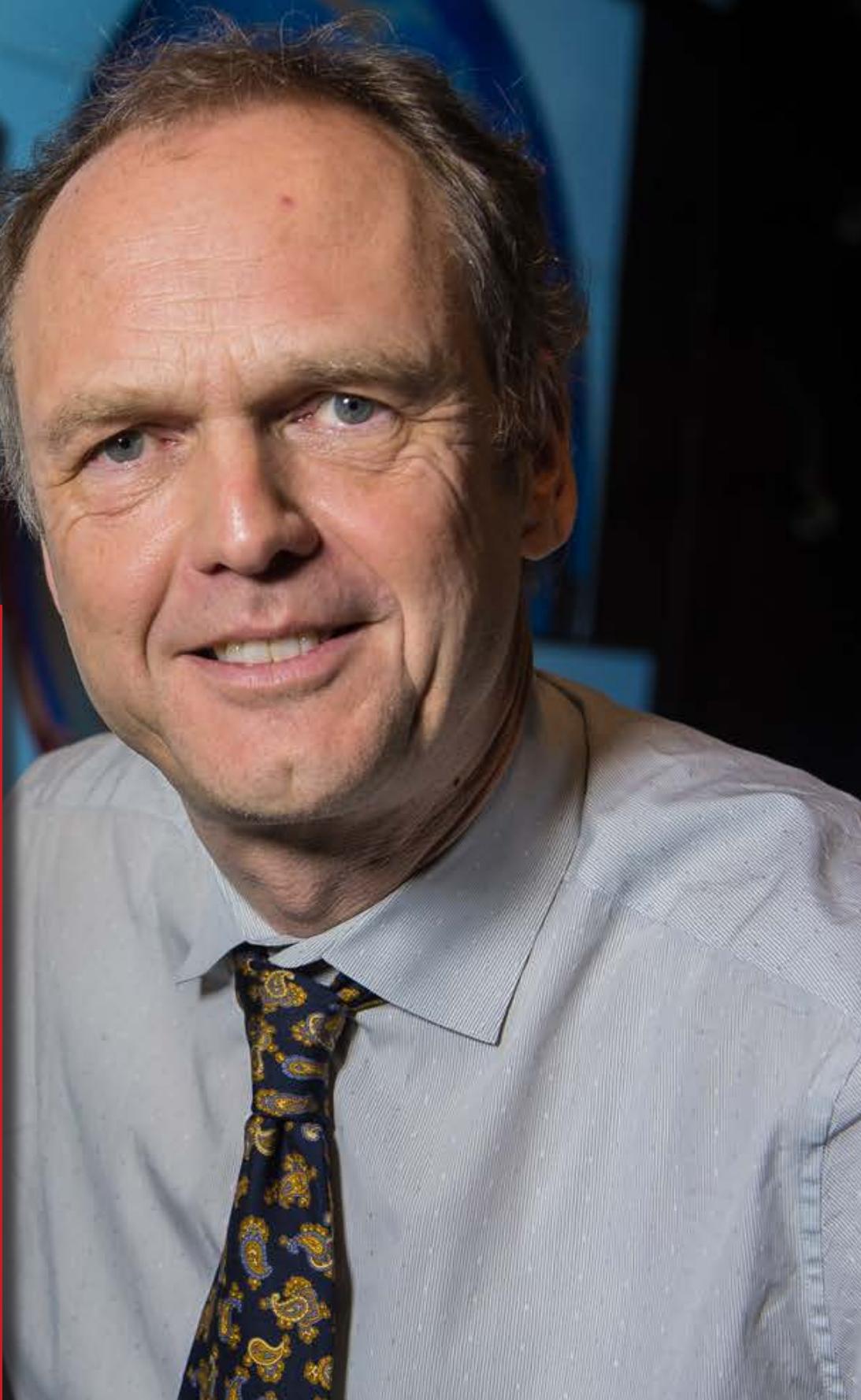
SIMONS FOUNDATION

Sponsors also benefit from:

- Significant additional funding we bring to bear from federal, international and foundation sources;
- Our communications network: our events calendar, email-list and news announcements keep our sponsors abreast of advances in our fields;
- Our location in Houston: our biweekly GMIG seminar and Departmental colloquium series are accessible to local collaborators.



Research Update for 2018: GMIG has made fundamental progress in the study of seismic inverse problems with anisotropic and nonlinear elasticity. We now have a full understanding of how to conditionally recover anisotropy from time-domain data, either via particular parameterizations on (unknown) subanalytic sets or, in the smooth case, exploiting techniques from Finsler geometry (revisiting tomography and using the boundary distance function as the data, while allowing the formation of caustics). We proved that nonlinear interaction of polarized (distorted plane) waves opens new ways of inversion and are pursuing experiments to demonstrate this phenomenon in the laboratory. We generalized our scattering control and reconstruction from scalar to elastic waves, assuming isotropy. Here, the disentangling of internal multiple scattering must be tied to the reconstruction. We developed computational methodologies to study complex faults and seismicity, introducing viscosity solutions, and established a first uniqueness result for the coseismic deformation inverse problems recovering the fault shape and dislocation from local GPS or InSAR data. We developed a new matrix-free iterative algorithm for solving the 3D Helmholtz equation at high frequencies employing contour integration and pseudo-time stepping. Finally, we have been constructing the first deep neural networks, interpretable as these fully represent direct nonlinear and iterative reconstructions.

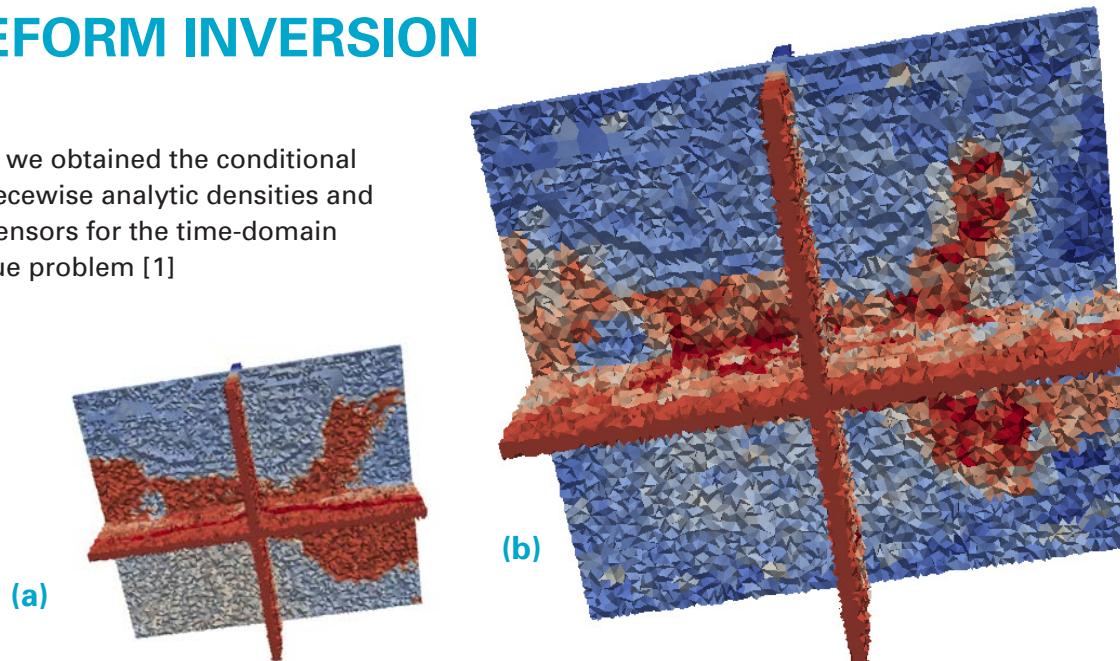


The research of GMIG is grouped into interconnected categories having inverse problems, multi-scale and scale coupling, deep learning, fast algorithms and massively parallel computing in seismology, geodesy and electromagnetics as common themes.

- FULL WAVEFORM INVERSION**
- SCATTERING CONTROL,
DIRECT INVERSE METHODS**
- DEEP LEARNING & INVERSE
PROBLEMS**
- GEOMETRIC INVERSE
PROBLEMS, ANISOTROPY**
- FAST ALGORITHMS, MASSIVELY
PARALLEL COMPUTING**
- NONLINEAR ELASTIC WAVE
INTERACTION**
- FAULTS, INVERSE DISLOCATION
AND INDUCED SEISMICITY**
- RELAXATION, ATTENUATION**

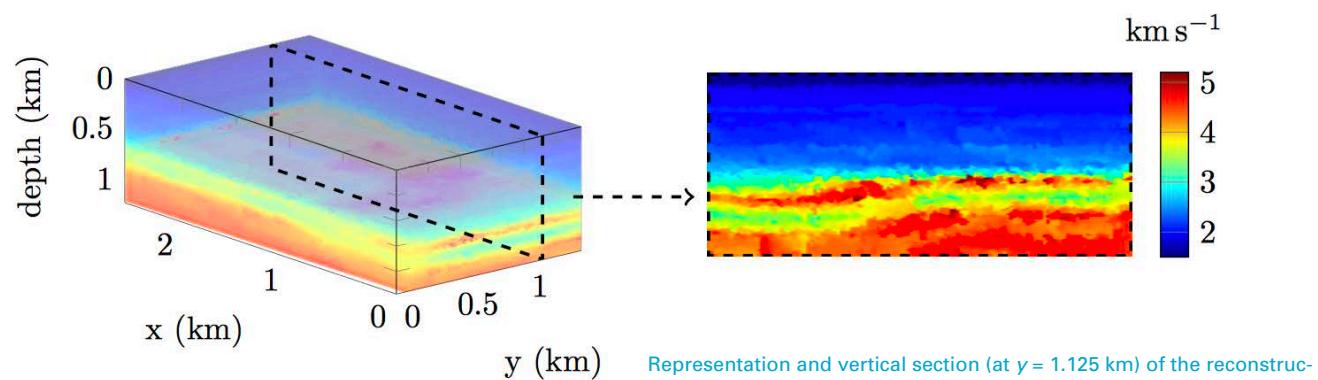
FULL WAVEFORM INVERSION

In recent work in FWI, we obtained the conditional unique recovery of piecewise analytic densities and anisotropic stiffness tensors for the time-domain inverse boundary value problem [1]



Multi-parameter elastic full-waveform inversion with rigorous iterative regularization from multi-frequency vibroseis data: reconstruction of subsurface elastic parameters for a complex salt problem (SEG SEAM phase 1 model) using unstructured tetrahedral mesh. True model (a) and (b) inverted Vp model. (2018) J. Shi, E. Beretta, M. V. de Hoop, E. Francini, S. Vessella. In Preparation.

We obtained a Lipschitz stability result for the inverse boundary value problem associated with the Helmholtz equation using partial Cauchy data, for highly adaptive piecewise linear wave speed representations on a given domain partition. Such a partition is closely connected to segmentation. Cauchy data are assimilated in dual-sensor acquisition and avoid the presence of eigenfrequencies in our earlier analysis using the Dirichlet-to-Neumann map as the data [2].

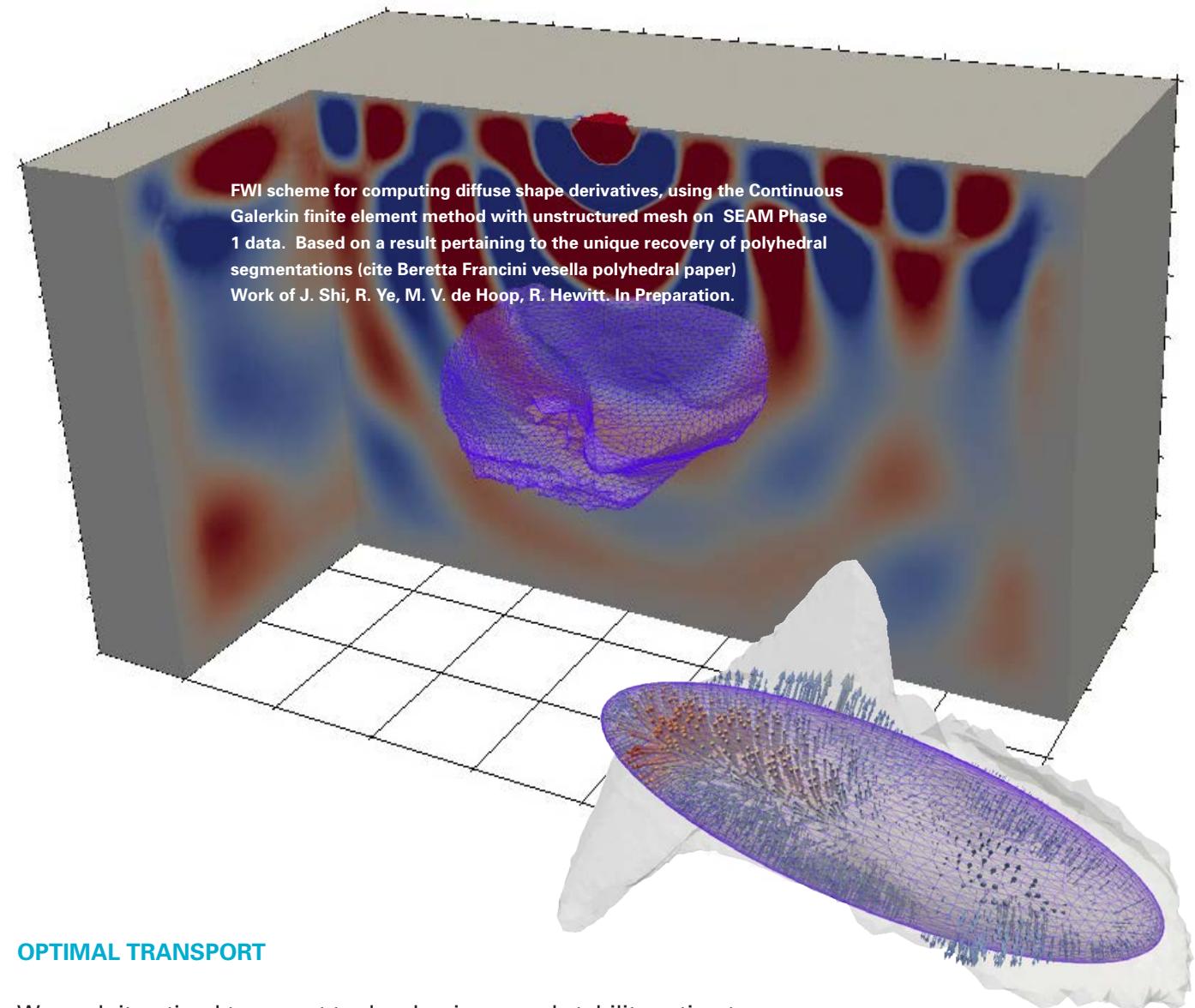


Representation and vertical section (at $y = 1.125$ km) of the reconstruction of a model from 10Hz Cauchy data after 175 iterations, where the sources in the acquisition are positioned in a three-dimensional area [2].

[1] "Unique recovery of piecewise analytic densities and stiffness tensors from the elastic-wave Dirichlet-to-Neumann map" (2018) M. V. de Hoop, G. Nakamura, J. Zhai. arXiv:1803.01091

[2] "Inverse problem for the Helmholtz equation with Cauchy data: Reconstruction with conditional well-posedness driven iterative regularization" (2017) G. Alessandrini, M. V. de Hoop, F. Faucher, R. Gaburro and E. Sincich. arXiv:1712.00398

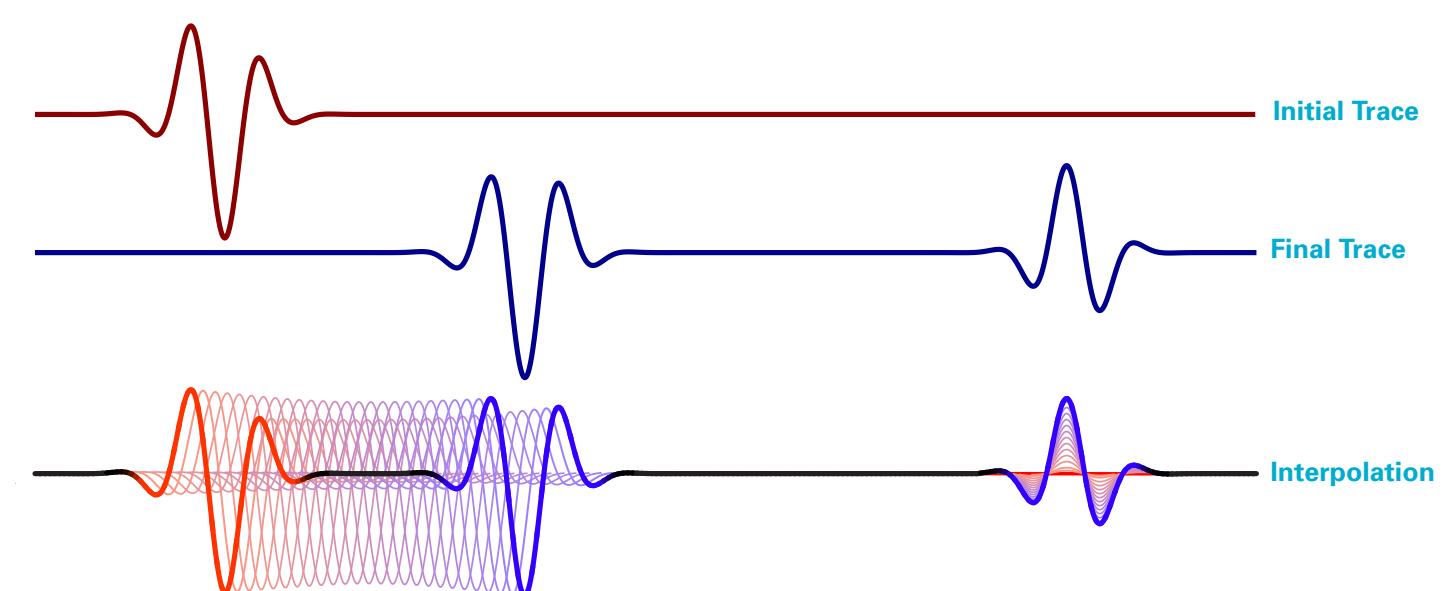
[3] "Enhanced stability for the initial boundary value problem for the wave equation with optimal transport" (2018) P. Caday, M. V. de Hoop, In Preparation.



FWI scheme for computing diffuse shape derivatives, using the Continuous Galerkin finite element method with unstructured mesh on SEAM Phase 1 data. Based on a result pertaining to the unique recovery of polyhedral segmentations (cite Beretta Francini vesella polyhedral paper)
Work of J. Shi, R. Ye, M. V. de Hoop, R. Hewitt. In Preparation.

OPTIMAL TRANSPORT

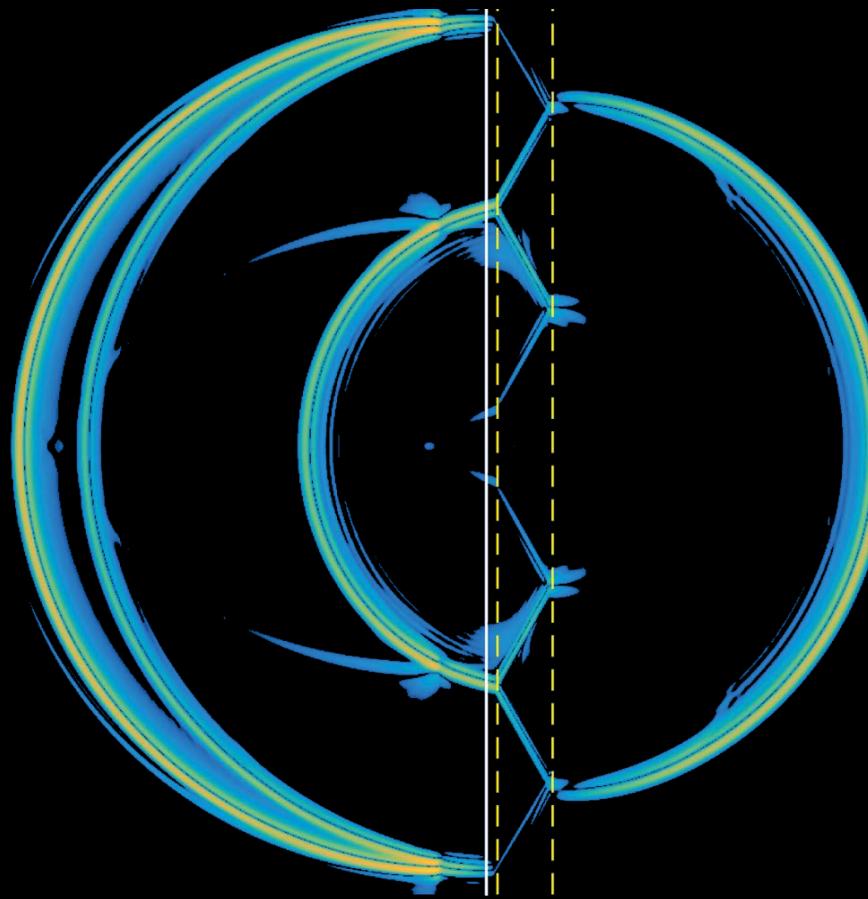
We exploit optimal transport to develop improved stability estimates for the scalar-wave inverse boundary value problem reminiscent of their geometric inverse problem counterparts; and introduce a new unbalanced extension of the Kantorovich-Fisher-Rao distance [3].



SCATTERING CONTROL, DIRECT INVERSE METHODS

We obtained a fundamental uniqueness result for the inverse boundary problem associated with the wave equation with piecewise smooth wavespeeds. The scattering control allows for disentangling of internal multiple scattering without knowledge of the wave speed or interfaces, and is implemented through instantaneous time mirrors [1].

We then developed a scattering-control-based approach to establish the unique recovery of piecewise smooth P - and S -wave speeds under an appropriate foliation condition. Here, we make use of techniques from microlocal analysis while the proof is essentially constructive [2].



[1] "Scattering control for the wave equation with unknown wave speed" (2017) P. Caday, M. V. de Hoop, V. Katsnelson, G. Uhlmann. Arch. Ration. Mech. Anal. In Press.

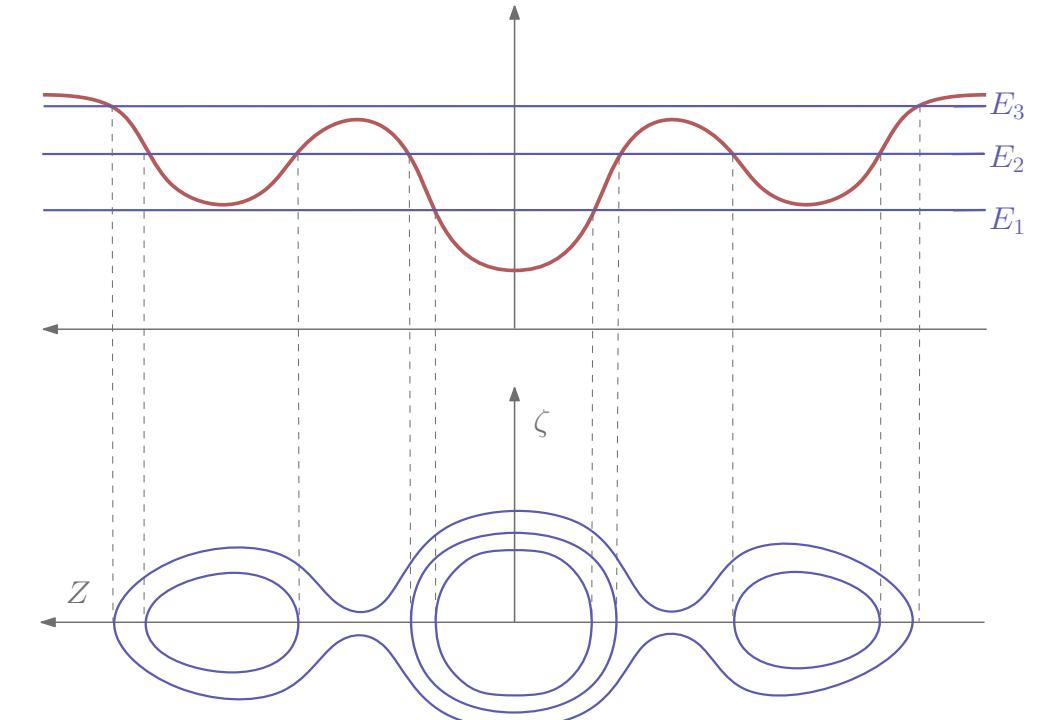
[2] "Reconstruction of piecewise smooth wave speeds using multiple scattering" (2018) P. Caday, M. V. de Hoop, V. Katsnelson, G. Uhlmann. Trans. Amer. Math. Soc. In Press.

SPECTRAL INVERSE PROBLEMS

For the inverse problem for the ``shallow'' subsurface involves elastic surface waves, we completed the semiclassical analysis of Love and Rayleigh waves and surface waves in generally anisotropic media [1]. We obtained Weyl's formulas for counting surface-wave modes. We have now analyzed the inverse problem for Love waves and obtained a uniqueness result for a smooth wave speed exhibiting multiple minima and maxima in depth from the semiclassical spectrum, that is, high-frequency phase velocities, alone. We are continuing with the corresponding analysis for Rayleigh waves. There is a natural connection with Earth's normal modes. Concerning these modes, we established a spectral rigidity result for the round Earth subject to Herglotz condition through a trace formula [2].

[1] "Semiclassical analysis of elastic surface waves" (2018) M. V. de Hoop, A. Iantchenko, G. Nakamura, J. Zhai. arXiv:1709.06521

[2] "Spectral rigidity for spherically symmetric manifolds with boundary" (2017) M. V. de Hoop, J. Ilmavirta, V. Katsnelson. arXiv:1705.10434



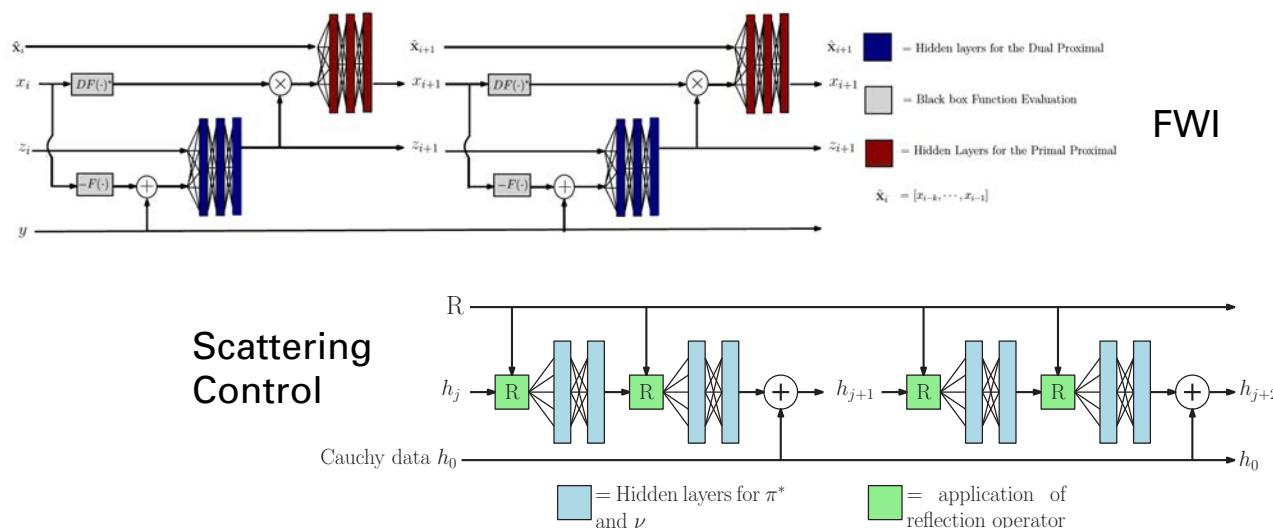
Geometric considerations: recovery of S -wave speeds with surface waves

DEEP LEARNING & INVERSE PROBLEMS

The applicability of neural networks to geophysics-based problems holds great promise in revealing detail and reducing the complexity and cumbersome technique currently required to build accurate information models about the earth. Our goal is to demonstrate, both mathematically and computationally, the suitability of deep neural networks for learning to solve specific inverse problems of wave propagation through variable media [1, 2].

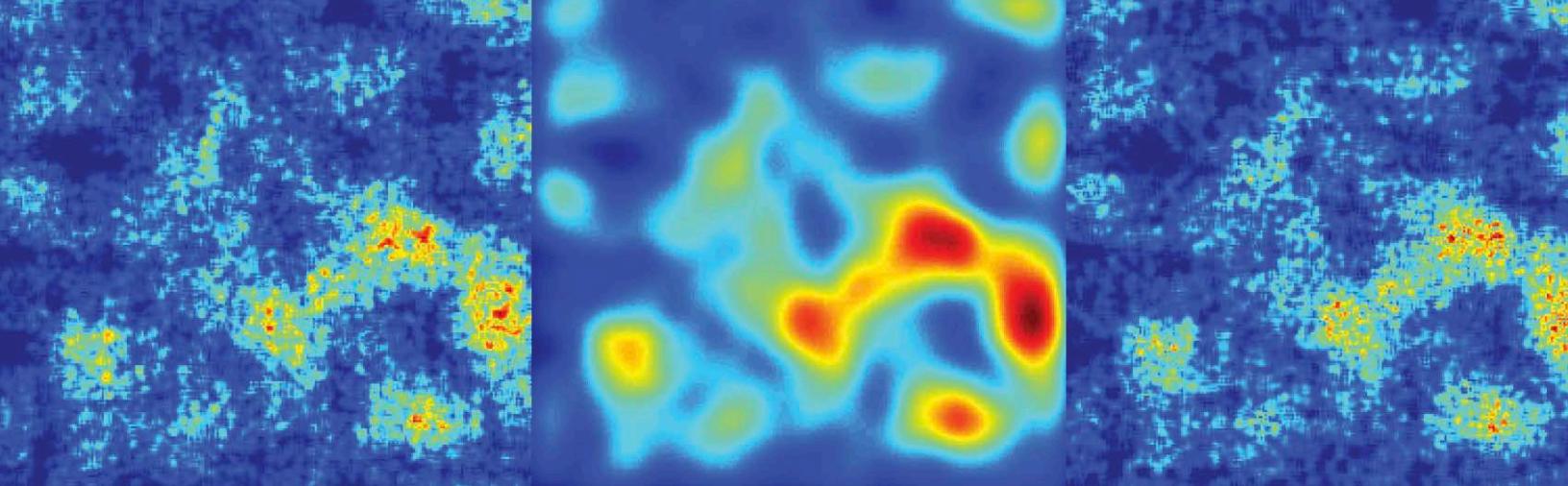
We have initiated a comprehensive research effort in the mathematical analysis of deep neural networks and their impact in *nonlinear* inverse problems. We have been focusing on deep architectures arising from direct nonlinear methods and by variational regularization, learned reconstruction, and data-driven learned regularization using feature spaces. Deep generative models replace the role of subspaces on which Lipschitz stability estimates hold. Work continues with an examination of deep learning mitigating ill posedness in inverse problems [3].

"Unrolled iterations" representations of deep learning architectures:

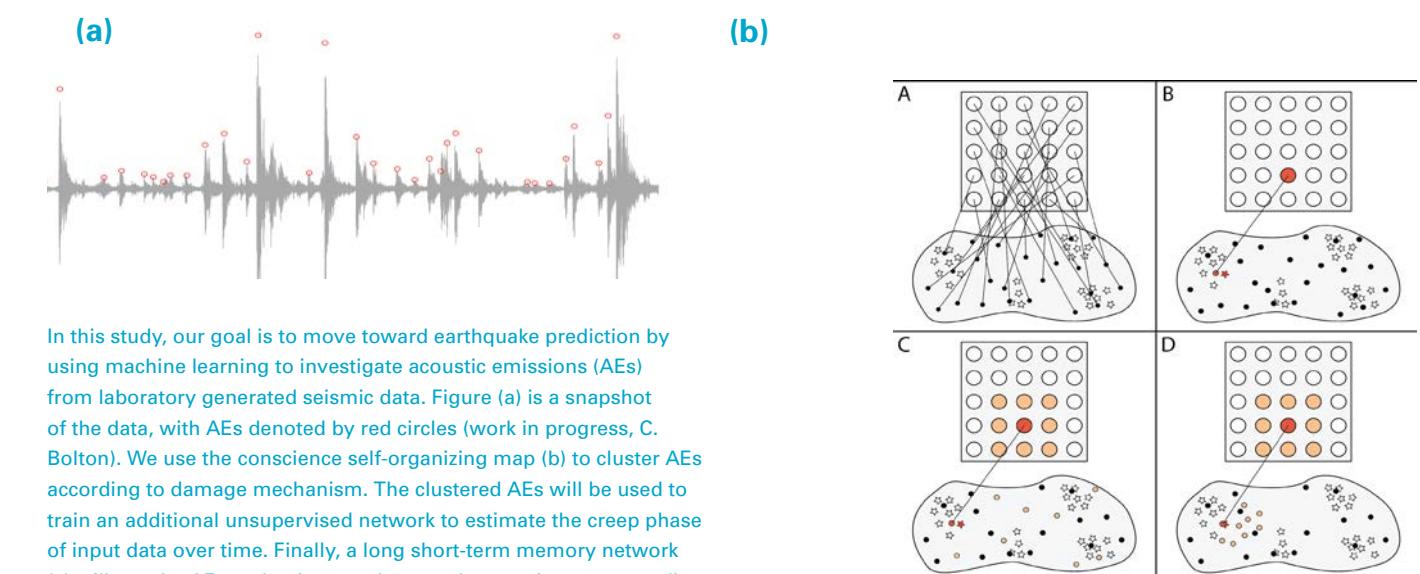


In the implementation and training of deep neural networks to inverse problems, prior mathematical understanding allows us to more deliberately choose appropriate learning strategies; a 'constructed' deep neural net is the result. Our work will investigate and create deep neural network architectures specifically applicable to problems of geophysical imaging. The cross-fertilization between analysis of inverse problems and machine learning presents exciting opportunities to exploit machine learning's ability to capture multi-scale geometric aspects of data and extract signatures and physics from data not previously or traditionally used in building models.

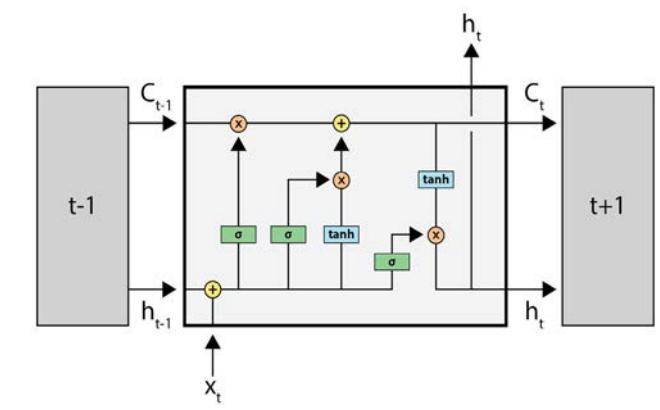
- [1] "Deep convolutional sufficient statistics for inverse problems" (2017) M. V. de Hoop, I. Dokmanic, J. Bruna, S. Mallat. GMIG Project Review, April 27-28, Rice University.
- [2] "Generalization and regularization in deep learning for nonlinear inverse problems" (2018) C. Wong, M. Lassas, M. V. de hoop. In Preparation.
- [3] "Deep learning mitigating ill-posedness in inverse problems" (2018) M. V. de Hoop, I. Dokmanic, J. Bruna, S. Mallat. GMIG Project Review, April 18-19, Rice University.



Deep generative models for signal disentanglement and pseudolinearization of inverse problems with neural networks was explored [4, 5]. In an application, we trained deep neural networks to 'learn' scattering control by designing a DNN with the structure of an unrolled iterative method. This resulted in a trained neural network that learned, through repeated probing of the unknown medium, to modify Cauchy data with trailing pulses that eliminate multiple reflections in the one-dimensional setting, mimicking scattering control [6].



- [4] "Deep generative models for geophysical signal disentanglement" (2018) T. Nguyen. GMIG Project Review, April 18-19, Rice University.
- [5] "Optimization with neural network preconditioners" (2018) P. Caday. GMIG Project Review, April 18-19, Rice University.
- [6] "A neural network architecture learning scattering control" (2018) P. Caday, J. Cocola, M. V. de Hoop, C. A. Wong. GMIG Project Report, Vol 3:13 pp. 279-288.
- [7] Work in progress, H. Jasperson, C. Bolton, P. Johnson, C. Marone, M. V. de Hoop (2018).

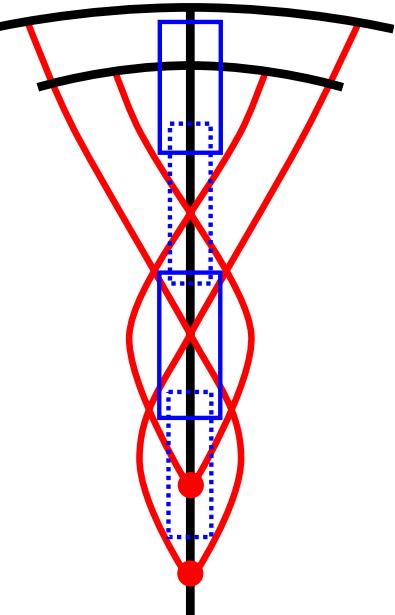
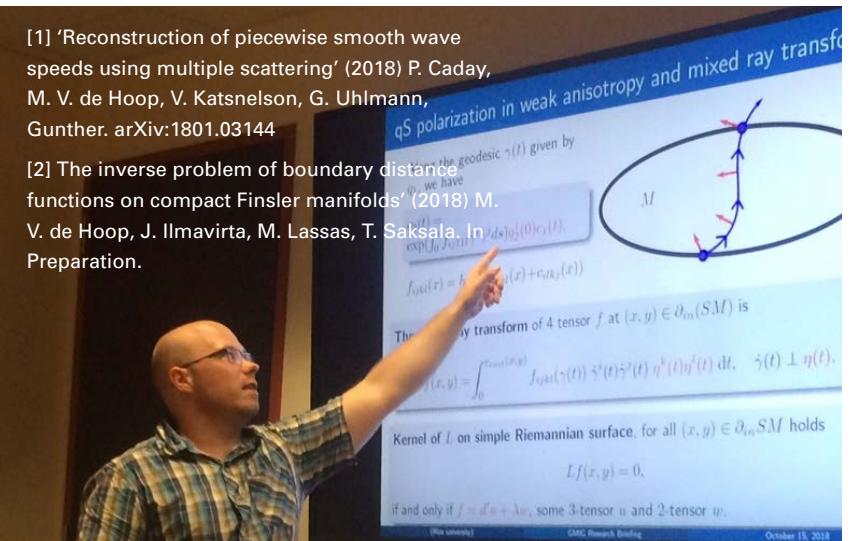


GEOMETRIC INVERSE PROBLEMS

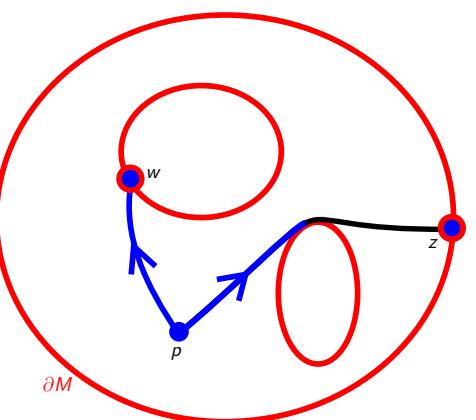
We obtained the injectivity of the local ("diving") geodesic ray transform for rank-4 tensors up to their solonoidal parts on Riemannian manifolds which captures travel-time tomography for qP waves in weakly anisotropic (relative to "elliptic") media. The result makes use of Melrose's scattering calculus for pseudodifferential operators [1]. We proceeded with studying anisotropy through Finsler geometry. Our first, and most fundamental result on recovery makes use of the boundary distance function as the data [2]. In the near future, we will extend this analysis to rigidity of broken geodesic flows using a scattering-type relation as the data. We analyzed the exploitation of interior point sources, in the context of seismicity, for the reconstruction of a Riemannian manifold [3].

We analyzed the broken ray transform with and without periodicity on spherically symmetric manifolds with piecewise $\{1,1\}$ metrics. This can be viewed as an analogue for ray-based velocity analysis with surface-related multiples. We established injectivity results under the Herglotz condition. To make these problems tractable in low regularity, we introduced and studied a class of generalized Abel transforms. To enable (fast) ray-based extended imaging and velocity inversion through extrapolation we developed higher-order Hamilton-Jacobi perturbation equations for anisotropic media [4]. We are currently completing the analysis of generalized Dix's anisotropic elastic inverse problem both for qP and qS , which we had solved in the isotropic scalar case with Riemannian geometry a couple of years ago.

We geometrized elastic anisotropy leading to the inverse problem of recovering a Finsler manifold from certain sphere data in a given open subset of the manifold. We solved this problem locally in the neighborhood of any geodesic through the open set. Combining this result for different geodesics showed that these data uniquely determine the universal cover of the Finsler manifold [5]. Finsler geometry also opens a new way of studying time-domain or hyperbolic inverse boundary value problems in the case of smoothly varying stiffness tensors.



Reconstruction procedure in the case of conjugate points for recovery of the isometry type of a Riemannian manifold from local boundary diffraction travel times.



We model the travel time of the acoustic waves with Riemannian geometry (the distance between points near to each other is given by travel time of the fastest wave). We study the inverse problem of travel time difference functions, that is the difference of travel times from given interior point to pairs of boundary points. The travel time difference from p to w and p to z is the black part of the curve from p to z and can obtain both positive and negative values.

[3] 'Inverse problem of travel time difference functions on compact Riemannian manifolds with boundary' (2018) M. V. de Hoop, T. Saksala. arXiv:1807.02576

[4] 'Spectral rigidity for spherically symmetric manifolds with boundary' (2018) M. V. de Hoop, J. Ilmavirta, V. Katsnelson. arXiv:1705.10434

[5] 'Dix's inverse problem on elastic finsler manifolds' (2018) M. V. de Hoop, J. Ilmavirta, M. Lassas. In Preparation.

FAST ALGORITHMS AND LARGE SCALE COMPUTING

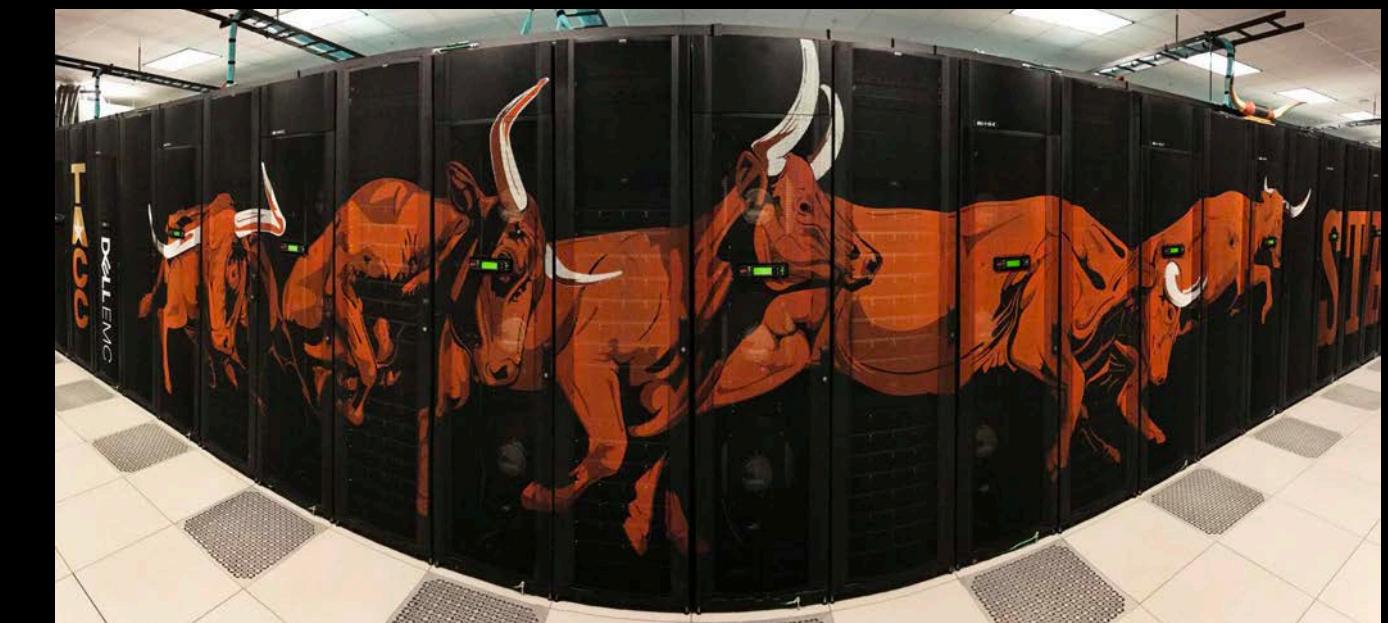
In our ongoing program on large-scale computing, we developed a new matrix-free iterative algorithm for solving the 3D Helmholtz equation at high frequencies employing contour integration and pseudo-time stepping. Here, dispersion compensation techniques enable fast convergence. We can now handle hundreds of wavelengths in each direction [1]. Furthermore, we completed a highly parallel polynomial filtering eigensolver [2]. The underlying technique is ideally suited for solving large-scale three-dimensional interior eigenvalue problems as it significantly enhances the memory and computational efficiency without loss of accuracy. The high efficiency and scalability of our approach were demonstrated on Stampede2 at the Texas Advanced Computing Center. Finally, we developed a method for analytical compression via proxy point selection and contour integration [1].

[1] "A matrix-free method for solving the 3D high-frequency Helmholtz equation based on contour integration and pseudo time-stepping" (2018) X. Liu, M. V. de Hoop. In Preparation.

[2] "Computing planetary interior normal modes with a highly parallel polynomial filtering Eigensolver" (2018) J. Shi, R. Li, Y. Xi, Y. Saad, M. V. de Hoop. SC '18: Proceedings of the International Conference for High Performance Computing, Networking, Storage, and Analysis.

COMPUTATIONAL RESOURCES

Researchers of GMIG have access to computational facilities at Rice University, and to nationally available resources via XSEDE (<https://www.xsede.org>), the NSF-funded virtual organization that integrates and coordinates the sharing of digital infrastructure for the US university research community. Our local Rice University resources include the NOTS and DAVinCI facilities, as well as the Rice Chevron Visualization facility. Our primary resource for large scale computing is the Stampede2 Supercomputer (<https://www.tacc.utexas.edu/systems/stampede2>) operated by the Texas Advanced Computing Center at the University of Austin, and accessed through XSEDE. We also frequently use Comet (http://www.sdsc.edu/support/user_guides/comet.html) at UC San Diego Supercomputer Center, and Pittsburgh Supercomputing Center's Bridges Large Memory Nodes: <https://www.psc.edu/bridges/user-guide/system-configuration>

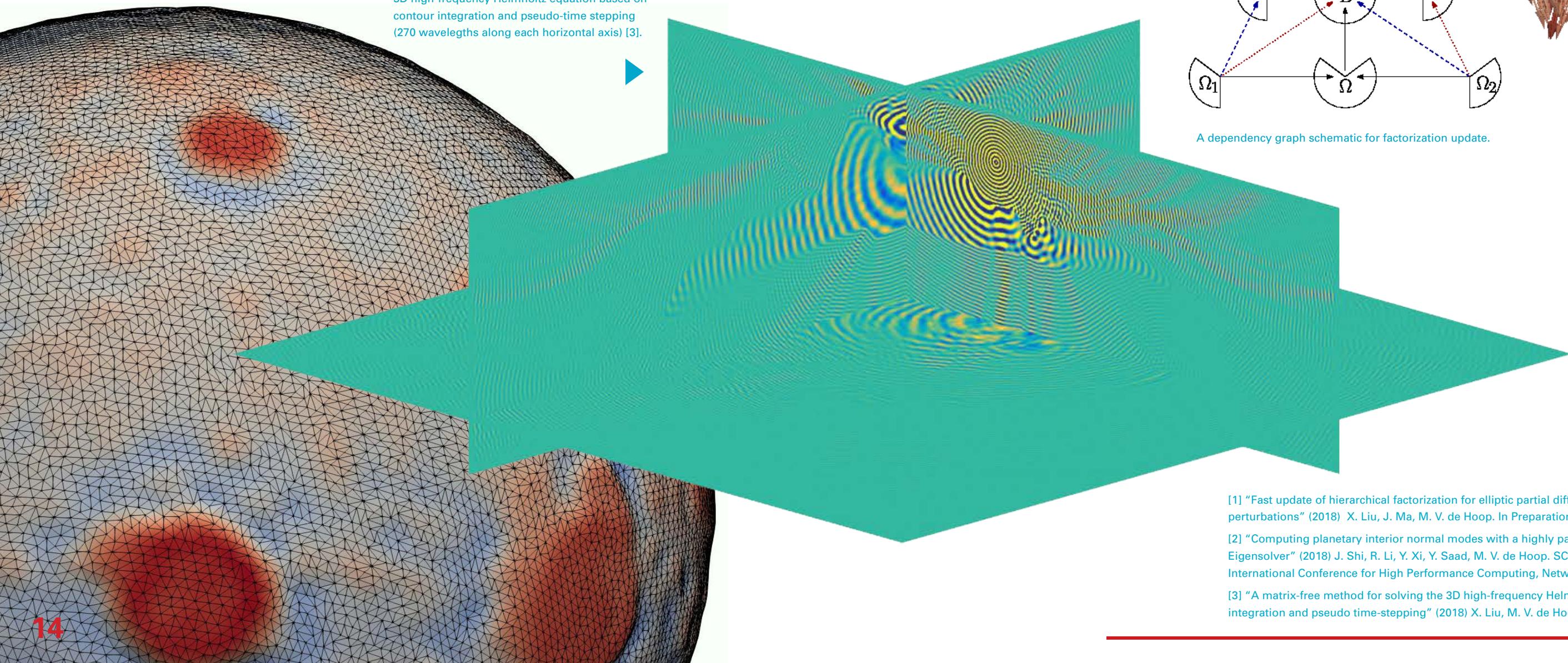


Stampede2 is the flagship supercomputer of The University of Texas at Austin's Advanced Computing Center (TACC). A strategic national resource, Stampede2 entered production in Fall 2017 as an 18 petaflop system featuring 4,200 Knights Landing (KNL) nodes and 1,736 Intel Xeon Skylake nodes. Stampede2 was deployed in partnership with Dell Inc, Intel Corporation, and Cray Inc, with the support of the National Science Foundation through award ACI-1540931. Photo reprinted by kind permission of TACC.

FAST ALGORITHMS AND LARGE SCALE COMPUTING (cont)

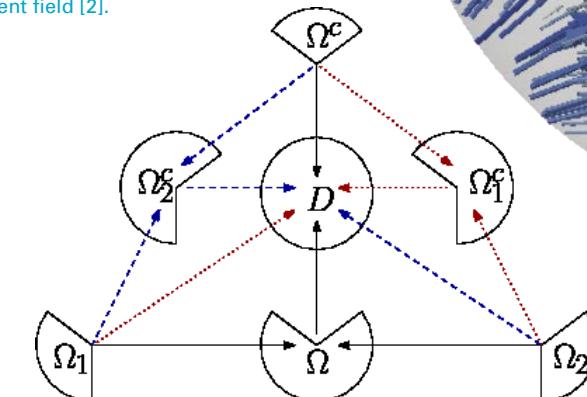
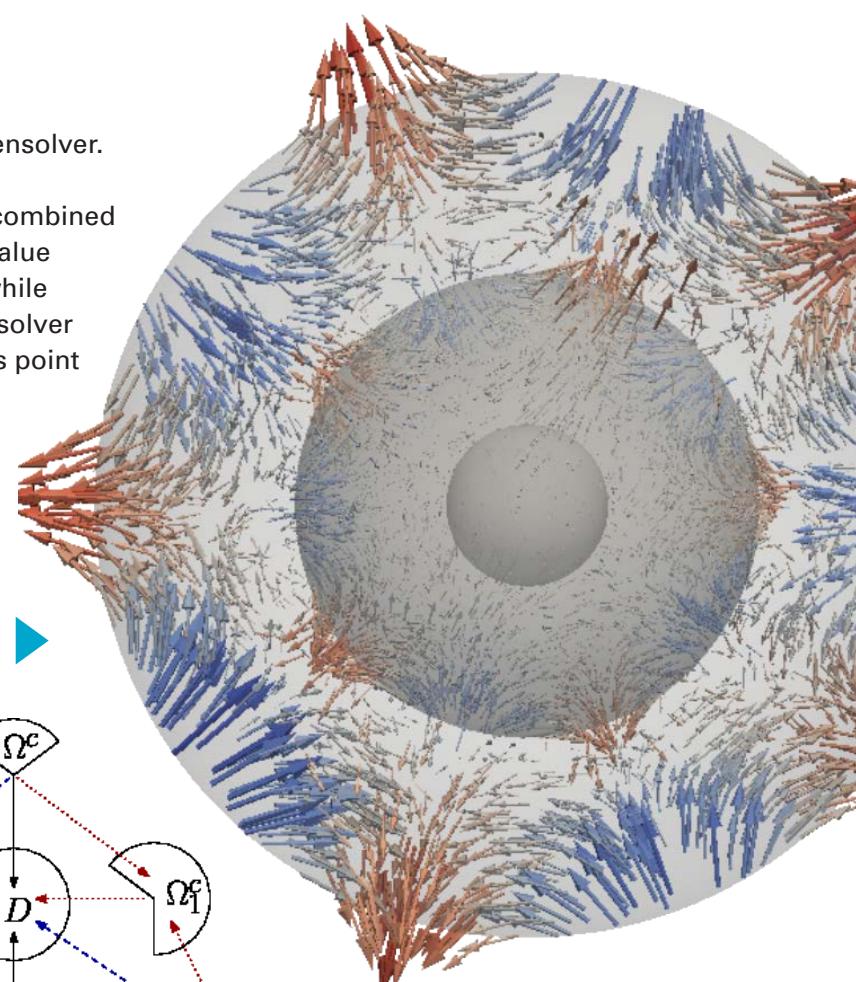
We developed a fast update of hierarchical factorization for elliptic partial differential operators with localized perturbations. This result is a breakthrough in improving the efficiency of time-harmonic FWI, for example, for updating a salt body. Based on a reference operator, a pre-computation is needed for a tree-based domain partitioning, a post-order factorization of interior subproblems, and a pre-order extraction of exterior subproblems. The pre-computation is fixed for different locations of perturbations, and is effective as long as the region of perturbations is intact in high-level domain partitioning. Furthermore, we completed the development of two interconnected hierarchical structures. The first one is a hierarchy of subdomains, separated by interfaces, where the interactions are communicated through dense subproblems. The second one is the hierarchically semiseparable (HSS) representation for each subproblem. Unlike existing rank-structured approaches, our technique is designed to eliminate the compression overhead for generating off-diagonal bases of the structured representations. Based on an auxiliary Robin boundary condition, the HSS off-diagonal bases are shared for overlapping subdomains. We then designed a preconditioning of the high-frequency Helmholtz equation. To this end, we created artificial damping near the interfaces to reduce the rank of the HSS representations. We provided the theory to relate the degree of damping with the compressibility and the convergence property.

Solution of a matrix-free method for solving the 3D high-frequency Helmholtz equation based on contour integration and pseudo-time stepping (270 wavelengths along each horizontal axis) [3].



Finally, we developed a new massively parallel eigensolver. We introduced, in parallel, two non-restart Lanczos approaches with polynomial and rational filtering, combined with spectral slicing, for solving generalized eigenvalue problems designed to minimize the memory cost while obtaining extreme accuracy. We adapted this eigensolver to the computation of interior eigenvalues in Earth's point spectrum and normal modes and mitigated the presence of an essential spectrum due to the presence of a fluid outer core.

A depiction of our computation of the spheroidal planetary normal modes of the elastic-gravitational system, approximated via the mixed finite element method on unstructured tetrahedral meshes from a three million element earth model. Colors represent the radial components of the displacement field [2].



A dependency graph schematic for factorization update.

[1] "Fast update of hierarchical factorization for elliptic partial differential operators with localized perturbations" (2018) X. Liu, J. Ma, M. V. de Hoop. In Preparation.

[2] "Computing planetary interior normal modes with a highly parallel polynomial filtering Eigensolver" (2018) J. Shi, R. Li, Y. Xi, Y. Saad, M. V. de Hoop. SC '18: Proceedings of the International Conference for High Performance Computing, Networking, Storage, and Analysis.

[3] "A matrix-free method for solving the 3D high-frequency Helmholtz equation based on contour integration and pseudo time-stepping" (2018) X. Liu, M. V. de Hoop. In Preparation.

NONLINEAR WAVE INTERACTION, RELAXATION, ATTENUATION

In recent work, we have completed geometrization of general elastic anisotropy properly entangling parametrizations and coordinates and general Finsler-Dix procedure [1]. We have constructed a parametrix for the wave equation with relaxation (memory kernels), including anisotropy. Our construction makes use of microlocal analysis techniques, the Laplace transform in time, and MacCamy's trick, and generalizes the correspondence principle [2]. We are currently revisiting time reversal and developing a new approach to incorporating attenuation, through an error operator that can be designed to be a contraction using microlocal energy estimates, in RTM-based inverse scattering and FWI gradients [3]. As a next step, we will study the full-inverse boundary value problem with attenuation.

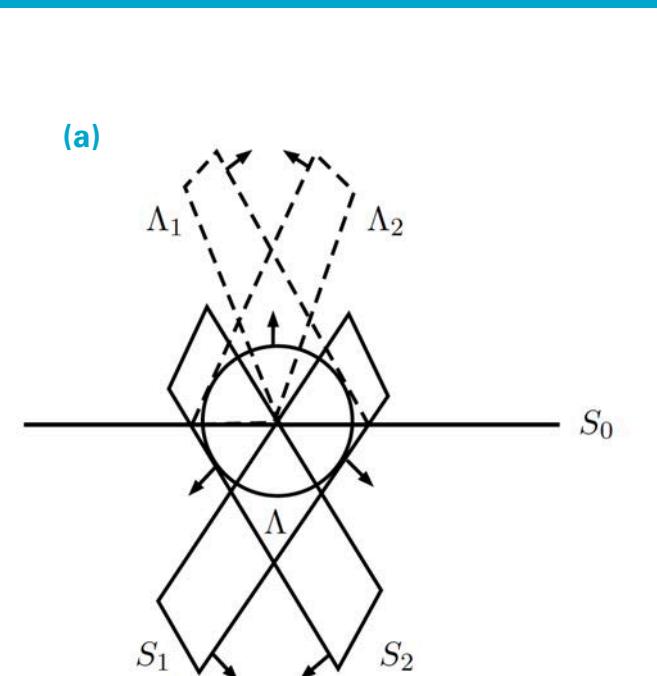
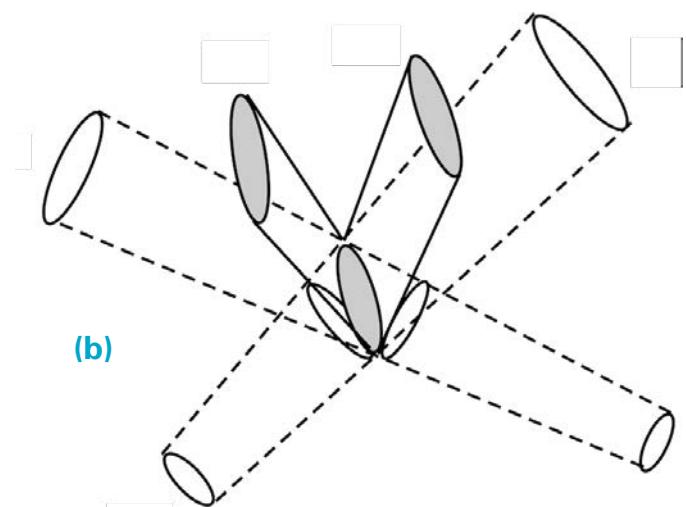
We analyzed, in the nonlinear acoustics case, the nonlinear interaction of distorted plane waves with discontinuities. In the nonlinear elastodynamics case, exploiting the presence of two distinct P and S metrics from the linear part, we analyzed the nonlinear interaction of polarized distorted plane waves. With these interactions, we studied various inverse problems yielding entirely new opportunities [4]. As a first step towards building a comprehensive framework of relaxation in inverse problems, we revisited the plane-wave analysis in infinite space. We are currently studying the unique continuation property for the wave equation with relaxation.

[1] "Geometrization of Dix's anisotropic elastic inverse problem" (2018)
M. V. de Hoop, J. Ilmavirta, M. Lassas. In Preparation.

[2] "Microlocal analysis of hyperbolic initial value problems with weak memory terms" (2017) J. Cocola, M. V. de Hoop. GMIG Project Report, Vol 2:12 pp. 251-269.

[3] "Microlocal compensation relaxation in RTM" (2018) J. Cocola.
GMIG Project Review, April 18-19, Rice University.

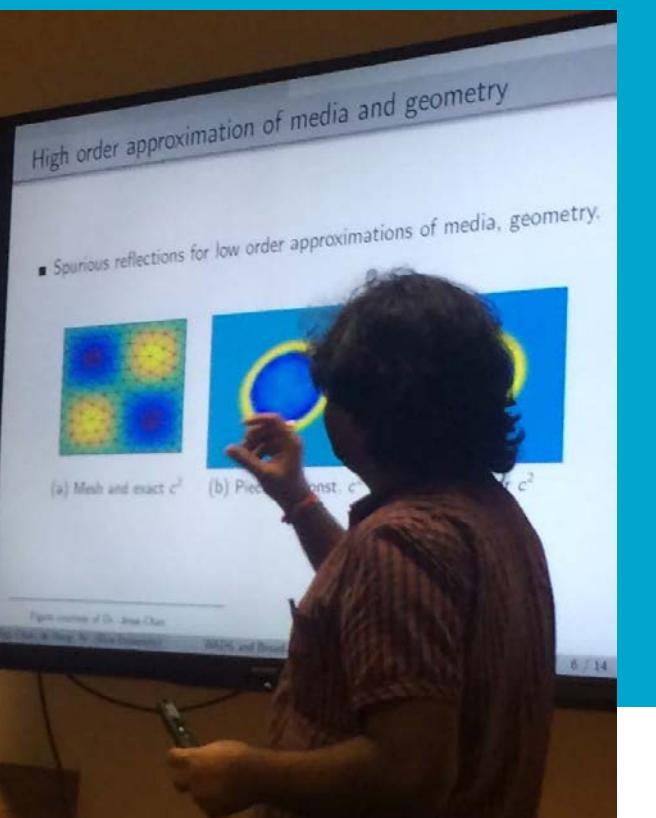
[4] "Nonlinear responses from the interaction of two progressing waves at an interface" (2018) M. V. de Hoop, G. Uhlmann, Y. Wang. Annales de l'Institut Henri Poincaré C / Analyse non linéaire, doi.org/10.1016/j.anihpc.2018.04.005



Nonlinear responses from the interaction of two progressing waves: (a) at an interface in the acoustic case and (b) polarizations in the elastic case. New waves are generated from the interactions. The incident waves and the nonlinear responses determine the location of the interface and information of the nonlinear properties of the media [4].

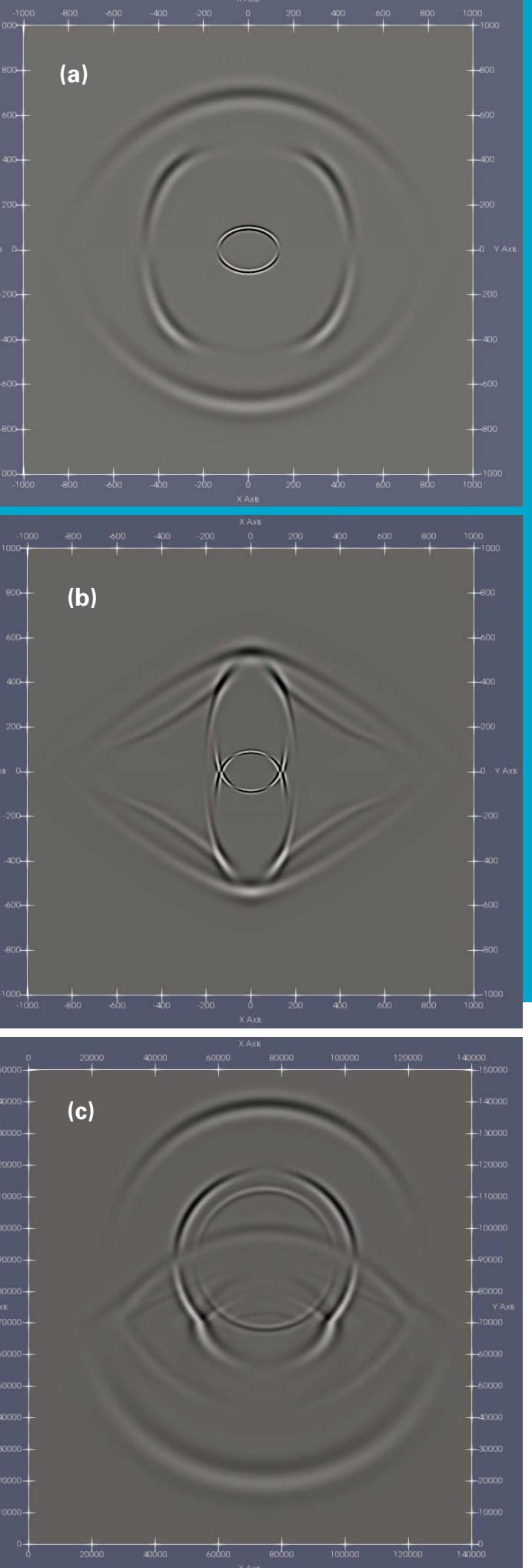
POROELASTICITY

We extended our DG approach to solving broad-band (Hz to kHz) Biot's equations including the dynamic permeability introduced by Johnson, Koplik and Dashen. We employed Strang's splitting to mitigate the stiffness resulting from the fluid's viscosity and developed a GPU implementation.



Images showing our recent numerical simulation of broad-band Biot's equation in low frequency regime (3.5 kHz) in an (a) orthotropic sandstone (b) epoxy-elsa, and (c) two layer interface model. Numerical solutions are obtained from weight adjusted discontinuous Galerkin method with a penalty flux. These images clearly represent the three modes of poroelastic waves i.e., (a) P wave, (b) S wave, and (c) slow P wave or Biot's mode. To resolve the slow P wave, it requires a very fine mesh discretization, which in these cases leads to 10×32768 ($N_p \times K$) dofs.

Work in progress, K. Shukla (2018).

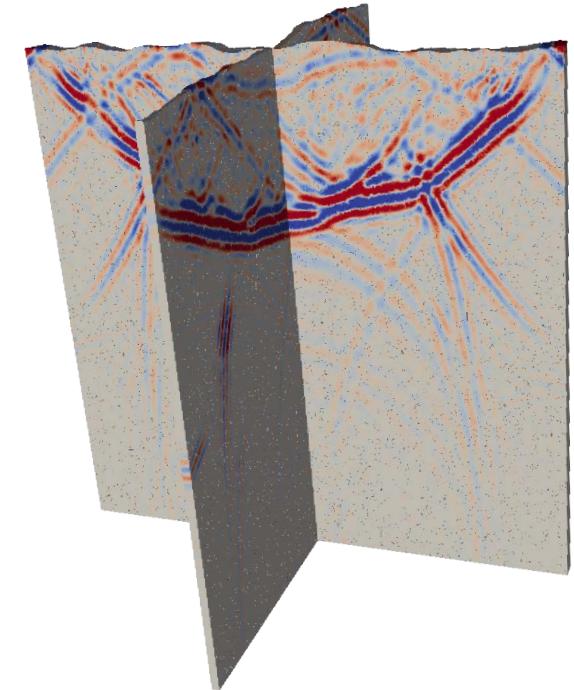
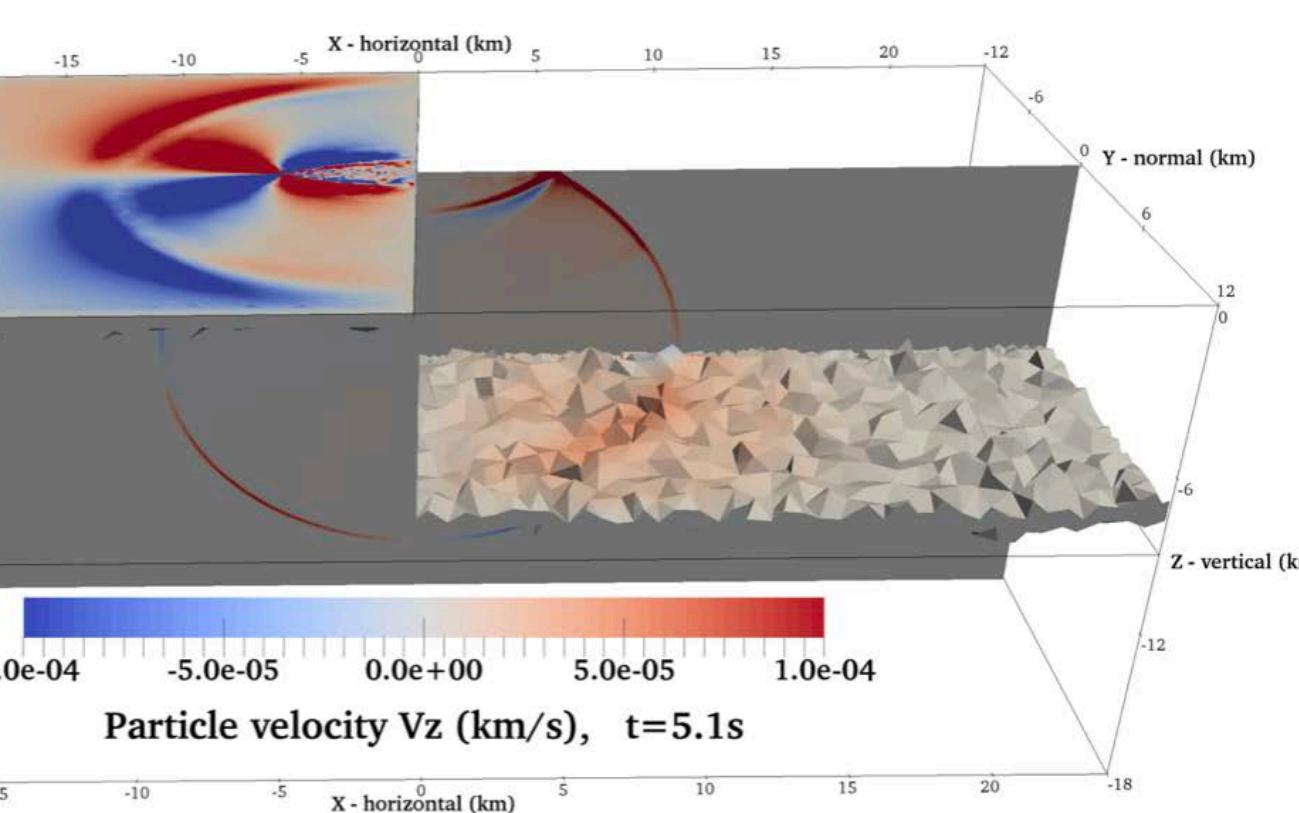
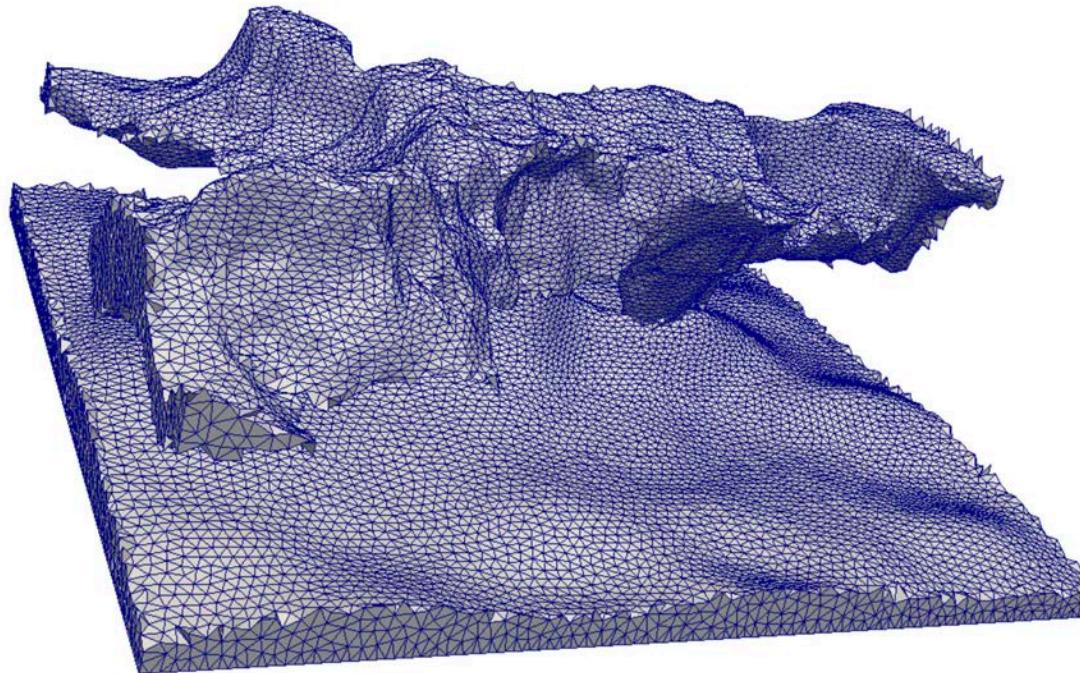


FAULTS, DISLOCATION AND INDUCED SEISMICITY

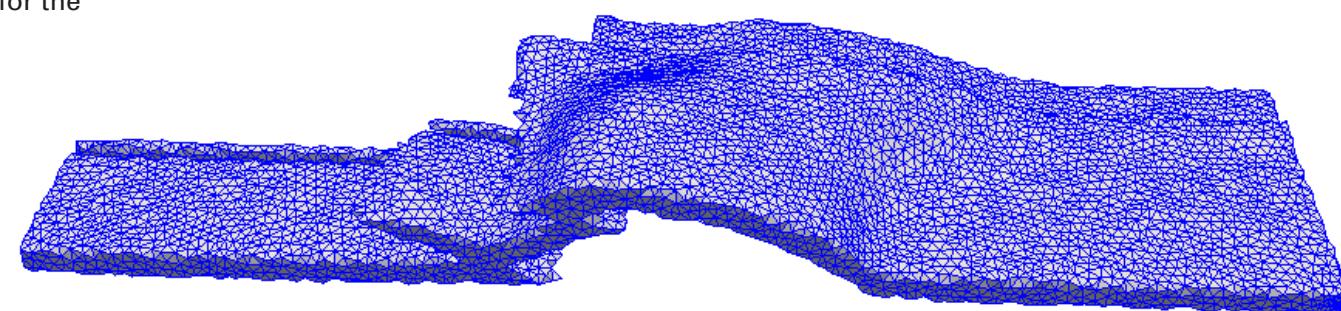
Planar rupture: snapshot of the components of particle velocity during simulation in three dimensions with unstructured tetrahedral mesh [1].



We embarked on analyzing the inverse problem appearing in geodesy to recover information about faults and dislocation in a heterogeneous environment. With appropriate assumptions, we obtained a uniqueness result. This is an integral part of studying forecasting of failure, that we are currently addressing with machine learning (see also p. 11). We completed a proper formulation of dynamic ruptures generating seismic waves in a self-gravitating planet through a multi-rate iterative coupling scheme necessitating the introduction of viscosity solutions [1]. We have been carrying out computational experiments rigorously investigating complex faults and seismicity for the first time.



The proposed multi-rate iterative coupling scheme, which is naturally discretized by a discontinuous Galerkin (DG) method, enables us to study the dynamic evolution of various physical systems at different scales. Notably, the nonlinear boundary conditions governed by rate- and state-dependent friction laws are imposed across the fault surface via DG numerical flux. The massively parallel implementation on the deformable meshes allows us to perform the realistic simulations for the dynamics on faults, dislocation and induced seismicity.



[1] "Analysis of dynamic ruptures generating seismic waves in a self-gravitating planet: A multi-rate iterative coupling scheme and well-posedness" (2018) R. Ye, K. Kumar, M. V. de Hoop. In Preparation.

[2] "Dynamic ruptures generating elastic waves in complex faults: well-posedness and stability of Discontinuous Galerkin method and iterative coupling scheme" (2018) R. Ye, M. Campillo, K. Kumar, M. V. de Hoop. GMIG Project Review, April 18-19, Rice University.



OUR COLLABORATIVE PROJECTS



Integral to the GMIG program are our inter-institutional collaborations with leading global research groups:

APPLIED HARMONIC ANALYSIS

Stéphane Jaffard (Université Paris-Est Créteil, France)
José Luis Romero (University of Vienna, Austria)

APPLIED PARTIAL DIFFERENTIAL EQUATIONS

Peter Markowich (KAUST, Saudi Arabia)

DEEP LEARNING

Joan Bruna (New York University – Courant Institute)
Ivan Dokmanić (University of Illinois at Urbana-Champaign)
Stéphane Mallat (ENS, Paris, France)
Ozan Öktem (KTH, Stockholm, Sweden)

EXPLORATION SEISMOLOGY

Einar Iversen (University of Bergen, Norway)
Hebert Montegranario (University of Antioquia, Colombia)
Milton Porsani (Universidade Federal da Bahia, Brazil)
Bjørn Ursin (NTNU, Norway)

GLOBAL SEISMOLOGY, GEODYNAMICS

Gabriele Cambiotti (University of Milan)
Michel Campillo (Université Grenoble Alpes, ISTERRE, France)
Flor de Lis Mancilla Pérez (University of Granada, Spain)
Heiner Igel (Ludwig Maximilians University Munich, Germany)
Paul Johnson (Los Alamos National Lab)
Maureen Long (Yale)
Laura Pyrak-Nolte (Purdue)
Robert van der Hilst (MIT)
Huajian Yao (University of Science and Technology of China)

INVERSE PROBLEMS

Giovanni Alessandrini (Università d. Studi di Trieste, Italy)
Elena Beretta (Politecnico di Milano, Italy)
Elisa Francini (University of Florence, Italy)
Alexei Iantchenko (Malmö University, Sweden)
Joonas Ilmavirta (University of Jyväskylä, Finland)
Victor Isakov (Wichita State University)
Shuichi Jimbo (Hokkaido University, Japan)
Matti Lassas (University of Helsinki, Finland)
Gen Nakamura (Hokkaido University, Japan)
Lauri Oksanen (UCL, England)
Otmar Scherzer (University of Vienna, Austria)
Gunther Uhlmann (University of Washington)
András Vasy (Stanford)
Sergio Vessella (University of Florence, Italy)

LARGE-SCALE COMPUTING

Kundan Kumar (University of Bergen, Norway)
Yousef Saad (University of Minnesota)
Huai Zhang (Chinese Academy of Sciences)

RANDOM MEDIA, CLUTTER, INTERFEROMETRY

Josselin Garnier (Ecole Polytechnique, France)
Knut Sølna (University of California at Irvine)

GEO-MATHEMATICAL IMAGING GROUP: OUR TEAM

The GMIG industry project was initiated in 2007 by Professor Maarten de Hoop with the broader purpose of advancing the understanding of our planet's subsurface and developing fundamentally new technologies in exploration seismology. Based at Rice University, GMIG's contributing faculty span the Departments of Mathematics, Electrical and Computer Engineering, Earth Environmental and Planetary Sciences, and Computational and Applied Mathematics.

Core GMIG research is primarily sponsored by the partner companies of the Geo-Mathematical Imaging Group, with additional complementary projects funded by the National Sciences Foundation and other sources. We are deeply indebted to the Simons Foundation for funding under the Math + X program which continues the advancement of our fundamental research into new frontiers.

In 2018 with the support of the Simons Foundation we hosted the Math + X Symposium on Data Science and Inverse Problems in Geophysics, which marked a pivotal move of our group into deep learning and geophysics intertwined collaborations.



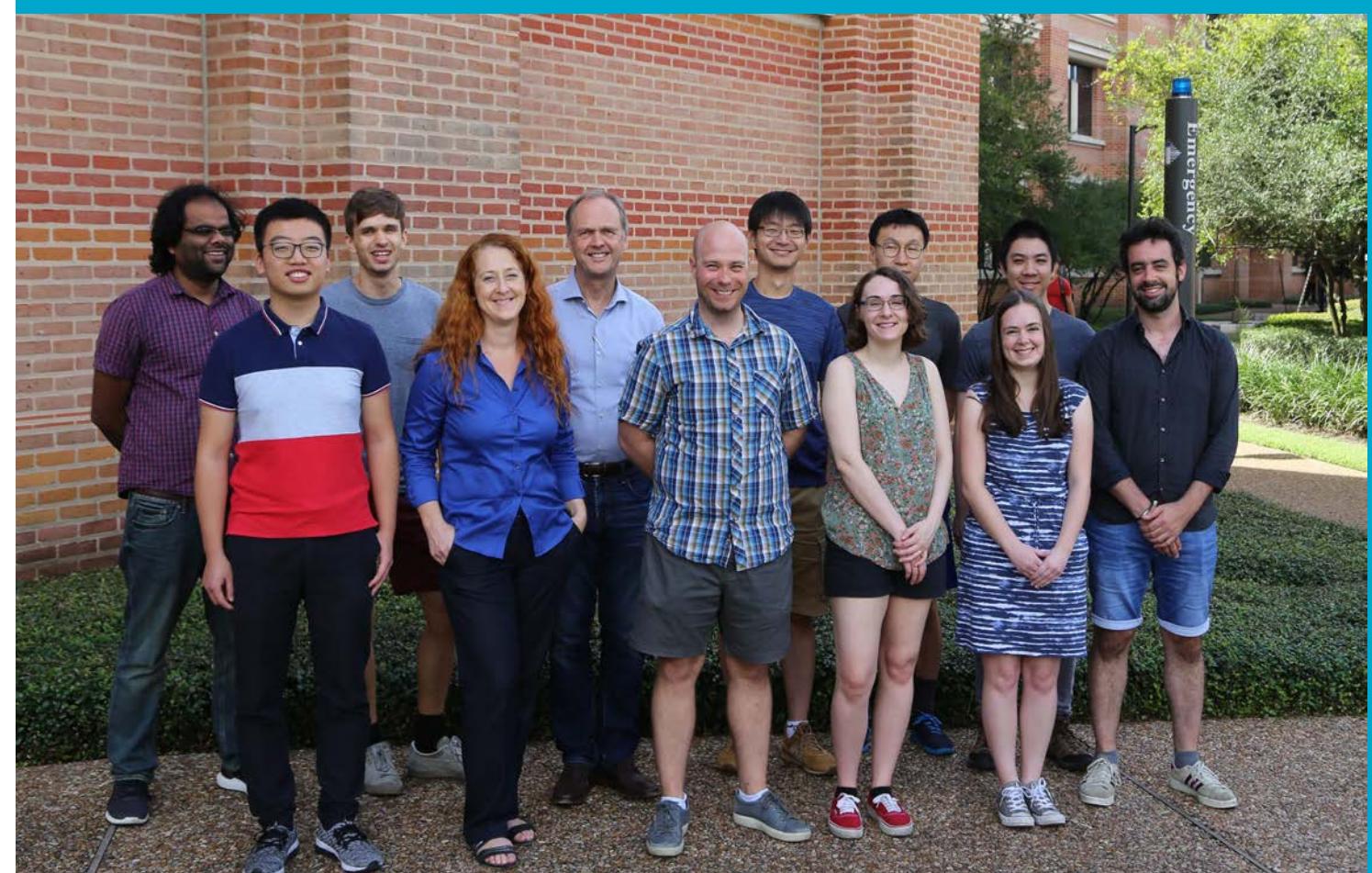
SIMONS FOUNDATION



MAARTEN V. DE HOOP

<http://maartendehoop.rice.edu/>

Maarten de Hoop is the Simons Chair in Computational and Applied Mathematics and Earth Science at Rice University. In addition to the Department of Computational and Applied Mathematics, he is appointed within the Math Department and the Department of Earth, Environmental and Planetary Sciences. His research interests are in inverse problems, microlocal analysis and computation, and applications in exploration and global seismology and geodynamics. He initiated GMIG while appointed at Purdue University (2007 – 2015) and has also previously been on faculty at Colorado School of Mines, Massachusetts Institute of Technology and the Graduate University of the Chinese Academy of Sciences Beijing (visiting faculty). He has served as senior research scientist and program leader with Schlumberger Gould Research Center and is currently scientific advisor (since 2010) with Corporate Science and Technology Projects, Total American Services, Inc. Prof. de Hoop received his Ph.D. in technical sciences from Delft University of Technology. In 1996 he was awarded the J. Clarence Karcher Award of the Society for Exploration Geophysicists.



Geo-Mathematical Imaging Group Fall 2018, Rice University, Houston:

(l to r) Khemraj Shukla, Visiting PhD student (Oklahoma State University); Jiayuan Han, PhD student (Rice Earth Sciences); Gregory Lyons, PhD student (Rice Mathematics); Miranda Joyce, GMIG staff; Professor Maarten de Hoop; Dr. Teemu Saksala, Simons Postdoctoral Fellow; Jia Shi, PhD student (Rice Earth Sciences); Hope Jasperson, PhD student (Rice Earth Sciences); Xiao Liu, PhD Student (Rice Computational and Applied Mathematics); Kate Begland, PhD student (Rice Earth Sciences); Dr. Christopher Wong, Postdoctoral Research Associate; Dr. Léonard Seydoux, visiting Postdoctoral Fellow (ISTerre, Université Grenoble Alpes).

GMIG's research team typically includes Graduate Students (primarily PhD), and Postdoctoral Fellows from Rice University's Departments of Computational and Applied Mathematics, Earth, Environmental and Planetary Sciences, Mathematics, and Electrical and Computer Engineering; and a steady flow of short- and long-term visiting students, faculty and collaborating scientists.

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'Unique recovery of a piecewise analytic density and stiffness tensor from the elastic-wave Dirichlet- to-Neumann map.' 2018. Maarten V. de Hoop, Gen Nakamura & Jian Zhai. In preparation.

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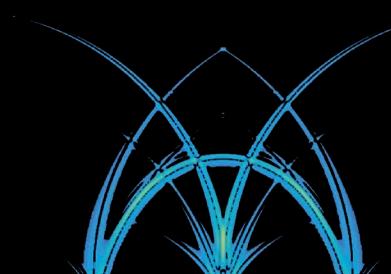
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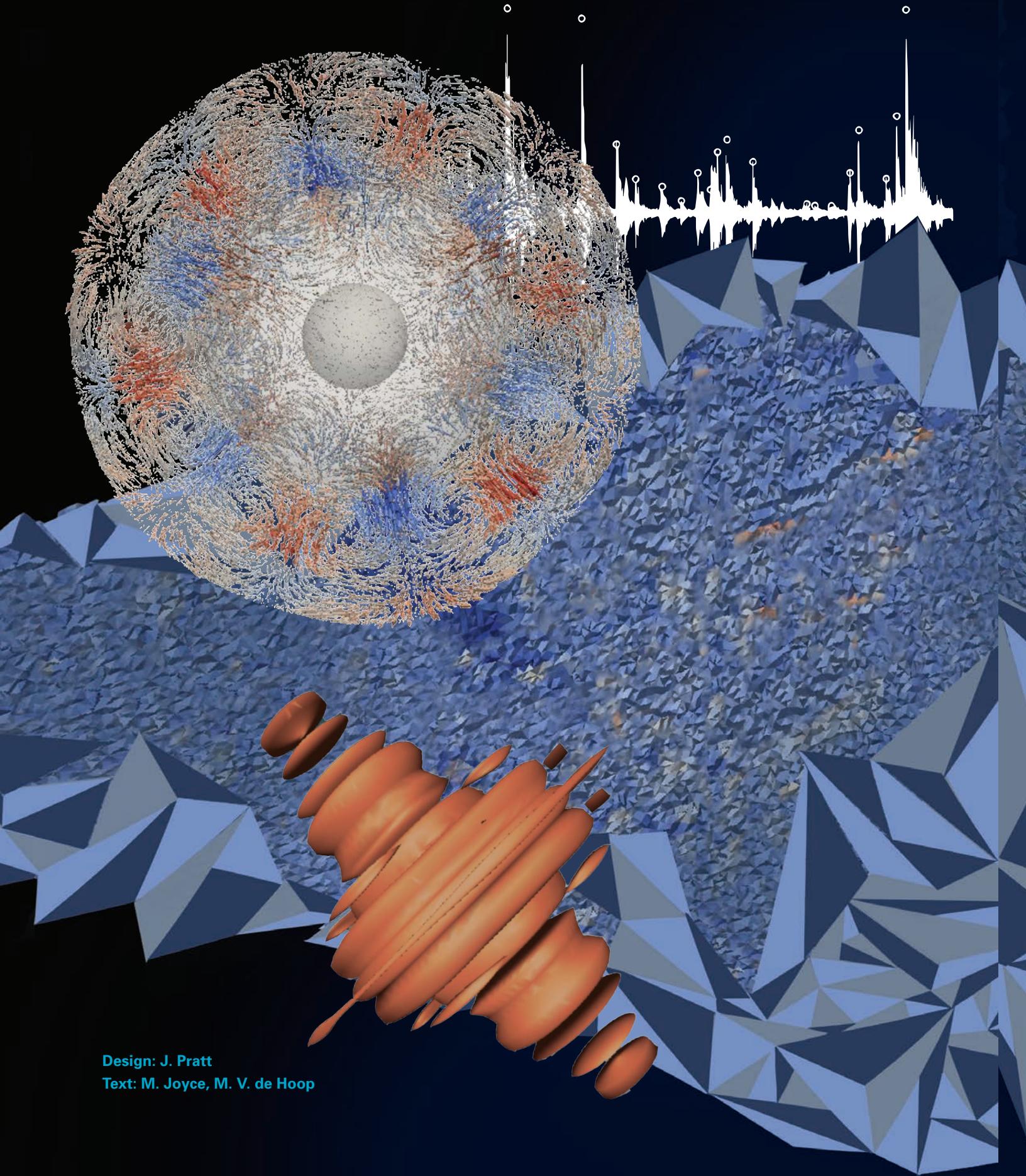
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